Rewrite $\operatorname{csch}(3\ln 2)$ in terms of exponential functions and simplify. **SCORE: ______ / 3 PTS**

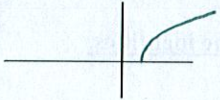
$$\boxed{\frac{2}{e^{3\ln 2}-e^{-3\ln 2}}} = \frac{2}{e^{\ln 2^3}-e^{\ln 2^{-3}}} = \boxed{\frac{2}{8-\frac{1}{8}}} \cdot \frac{8}{8} = \boxed{\frac{16}{63}}$$

1 POINT EACH

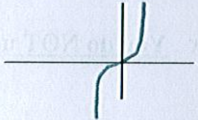
Sketch the general shape and position of the following graphs.
(Don't worry about specific $x-$ or $y-$ coordinates.) GRADED BY ME

**SCORE: ______ / 3 PTS**

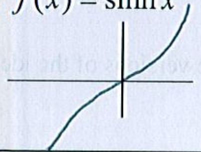
$f(x) = \cosh^{-1} x$



$f(x) = \tanh^{-1} x$



$f(x) = \sinh x$



Write and **prove** a formula for $\cosh(x+y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$. **SCORE: ______ / 6 PTS**

$$\boxed{\cosh x \cosh y + \sinh x \sinh y}$$

$$= \boxed{\frac{e^x+e^{-x}}{2} \cdot \frac{e^y+e^{-y}}{2}} + \boxed{\frac{e^x-e^{-x}}{2} \cdot \frac{e^y-e^{-y}}{2}}$$

$$= \boxed{\frac{e^{x+y} + \cancel{e^{x-y}} + \cancel{e^{-x+y}} + e^{-x-y}}{4}} + \boxed{\frac{e^{x+y} - \cancel{e^{x-y}} - \cancel{e^{-x+y}} + e^{-x-y}}{4}}$$

$$= \frac{2e^{x+y} + 2e^{-x-y}}{4}$$

$$= \boxed{\frac{e^{x+y} + e^{-(x+y)}}{2}}$$

$$= \cosh(x+y)$$

1 POINT EACH

Prove that $g(x) = \ln(x + \sqrt{x^2 - 1})$ is the inverse of $f(x) = \cosh x$ by simplifying $g(f(x))$. **SCORE: ______ / 5 PTS**

**You may use any identities that you found in part [1] of the Hyperbolic Functions Supplement without proving them.**

$\boxed{\ln(\cosh x + \sqrt{\cosh^2 x - 1})}$ 1

$= \ln(\cosh x + \sqrt{\sinh^2 x})$

$= \boxed{\ln(\cosh x + \sinh x)}$ $1\frac{1}{2}$

$= \boxed{\ln e^x}$ $1\frac{1}{2}$

$= \boxed{x}$ 1

There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry. **SCORE: ______ / 7 PTS**

[a] Write that identity involving $\sinh x$ and $\cosh x$. **<u>You do NOT need to prove the identity.</u>**

$$\boxed{\cosh^2 x - \sinh^2 x = 1} \quad 1$$

[b] Write any two of the three versions of the identity for $\cosh 2x$. **<u>You do NOT need to prove the identities.</u>**

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

OR

$$2\cosh^2 x - 1$$

OR

$$2\sinh^2 x + 1$$

1 POINT IF YOU GOT 1 IDENTITY

2 POINTS IF YOU GOT 2 (OR 3) IDENTITIES

[c] If $\coth x = 2$, find $\cosh x$ using identities.
**<u>You must explicitly show the use of the identities but you do NOT need to prove the identities.</u>**
**<u>Do NOT use inverse hyperbolic functions nor their logarithmic formulae in your solution.</u>**

$\frac{1}{2}$ $\tanh x = \frac{1}{\coth x} = \frac{1}{2}$

$1$ $\boxed{\operatorname{sech}^2 x = 1 - \tanh^2 x} = 1 - \frac{1}{4} = \boxed{\frac{3}{4}}$ $\frac{1}{2}$

$\frac{1}{2}$ $\boxed{\operatorname{sech} x = \frac{\sqrt{3}}{2}}$ (SINCE $\operatorname{sech} x > 0$ FOR ALL $x$) $\frac{1}{2}$

$\frac{1}{2}$ $\cosh x = \frac{1}{\operatorname{sech} x} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{2\sqrt{3}}{3}}$ $\frac{1}{2}$